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SOUND PRESSURE DISTRIBUTIONS AND RADIATION IMPEDANCE FOR FLEXIB--ETC(U)

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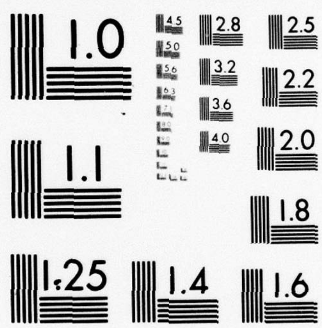

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SOUND PRESSURE DISTRIBUTIONS AND RADIATION IMPEDANCE FOR FLEXIBLE DISKS - PART III

(14) USL-912-79-62

by

(10) David T. Porter

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In two previous memoranda, (references (a) and (b)), methods were described for calculating the sound pressure on circular flexural disks and obtaining their radiation impedance. Unfortunately, it was only feasible to obtain the sound pressure by means of an electronic computer, and the radiation impedance data had to be obtained by numerically integrating this computer data for the sound pressure. In this memorandum the double integrals needed to compute the radiation impedance are evaluated. Simple expressions are given for radiation impedance suitable for direct hand calculation.

According to reference (a), radiation impedance for circular flexural disks, referred to their average normal surface velocity $\bar{u}$, is given as

$$Z = R + iX = \frac{1}{|\bar{u}|^2}\int_0^a P(r)\, u^*(r)\, 2\pi r\, dr. \qquad (1)$$

We take u to be

$$u = U e^{i\omega t} \sum_{n\ even} \alpha_n (r/a)^n = U e^{i\omega t}\left[\alpha_0 + \sum_{\substack{n=2\\ even}}^{\infty} \alpha_n (r/a)^n\right]. \qquad (2)$$

Most flexural disk problems of interest can be formulated using only even powers of r/a. Using the relation

$$P(r) = \rho c U e^{i\omega t} \sum_{n\ even} \alpha_n Z_n(r) \qquad (3)$$

it is seen that

$$R/\rho c \pi a^2 = 2\left(\frac{U}{|\bar{u}|}\right)^2 \int_0^1 \left(\mathrm{Re} \sum_{K\ even} \alpha_K Z_K\right)\left(\sum_{\ell\ even} \alpha_\ell x^\ell\right) x\, dx \qquad (4)$$

$(x = r/a)$

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and

$$X/\rho c\pi a^2 = 2\left(\frac{U^2}{|\bar{u}|^2}\right)\int_0^1\left(I_m \sum_{k\,even}\alpha_k Z_k\right)\left(\sum_{\ell\,even}\alpha_\ell x^\ell\right)x\,dx\,. \quad (5)$$

The pressure components Z_k are given in reference (a) for the case when k is 0, 2, and 4.

Interchanging integration and summation, we have

$$R/\rho c A = 2\left(\frac{U}{|\bar{u}|}\right)^2\sum_{k\,even}\sum_{\ell\,even}\alpha_k\alpha_\ell\int_0^1 R_e(Z_k)\,x^{\ell+1}\,dx \quad (6)$$

and

$$X/\rho c A = 2\left(\frac{U}{|\bar{u}|}\right)^2\sum_{k\,even}\sum_{\ell\,even}\alpha_k\alpha_\ell\int_0^1 I_m(Z_k)\,x^{\ell+1}\,dx\,. \quad (7)$$

From Equ. (2) we see that

$$|u| = U\sum_{n\,even}\alpha_n\,(r/a)^n \quad (8)$$

and

$$|\bar{u}| = \frac{1}{\pi a^2}\int_0^{2\pi}\int_0^a |u|\,r\,dr\,d\alpha = 2U\sum_{n\,even}\frac{\alpha_n}{n+2}\,. \quad (9)$$

It now remains to evaluate the following two integrals:

$$S_{k\ell} = \int_0^1 R_e(Z_k)\,x^{\ell+1}\,dx\,, \quad T_{k\ell} = \int_0^1 I_m(Z_k)\,x^{\ell+1}\,dx\,. \quad (10)$$

We also define the complex quantity

$$Q_{k\ell} = S_{k\ell} + i\,T_{k\ell} = \int_0^1 Z_k\,x^{\ell+1}\,dx \quad (11)$$

so that

$$Z/\rho c A = \frac{1}{2\left(\sum_{n\,even}\frac{\alpha_n}{n+2}\right)^2}\sum_{k\,even}\sum_{\ell\,even}\alpha_k\alpha_\ell Q_{k\ell}\,. \quad (12)$$

When k and ℓ are even, we shall be able to integrate $S_{k\ell}$ and $T_{k\ell}$. Two examples will be carried out, Q_{00} and Q_{20}. Tables I and II show $S_{k\ell}$ and $T_{k\ell}$ for k and ℓ = 0, 2 and 4. It will be shown in the Appendix that $Q_{k\ell} \equiv Q_{\ell k}$.

Referring to Equ. (11),

$$Q_{00} = \int_0^1 Z_0 \; x\,dx = \int_0^1 \left(1 - \frac{1}{\pi}\int_0^{\pi} e^{-ikR_0} d\alpha\right) x\,dx \tag{13}$$

$$= \frac{1}{2} - \frac{1}{\pi}\int_0^1 \int_0^{\pi} e^{-ikR_0} d\alpha \; x\,dx \; .$$

In reference (a), R_0 was defined as

$$R_0 = r\cos\alpha + (a^2 - r^2\sin^2\alpha)^{1/2} \tag{14}$$

so that

$$\frac{R_0}{a} = x\cos\alpha + (1 - x^2\sin^2\alpha)^{1/2} \; . \tag{15}$$

Therefore,

$$Q_{00} = \frac{1}{2} - \frac{1}{\pi}\int_0^1 \int_0^{\pi} e^{-ika\left(x\cos\alpha + \sqrt{1 - x^2\sin^2\alpha}\right)} x \; d\alpha\, dx \, . \tag{16}$$

In the x- α plane, the integration is over a semicircle with unit radius. We now make a change of variables

$$u = x\sin\alpha \, , \quad v = x\cos\alpha \tag{17}$$

and the differential area element x $d\alpha$ dx becomes du dv, and we have

$$Q_{00} = \frac{1}{2} - \frac{1}{\pi}\int_0^1 \int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} e^{-ika\left(v + \sqrt{1-u^2}\right)} dv\, du \tag{18}$$

$$= \frac{1}{2} - \frac{1}{\pi}\int_0^1 e^{-ika\sqrt{1-u^2}} \left(\int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} e^{-ikav} dv\right) du \; .$$

When the integration over v is performed, we have,

$$Q_{00} = \frac{1}{2} - \frac{1}{\pi}\int_0^1 e^{-ika\sqrt{1-u^2}} \left[\frac{e^{-ika\sqrt{1-u^2}} - e^{ika\sqrt{1-u^2}}}{-ika}\right] du$$

$$= \frac{1}{2} + \frac{i}{\pi ka} - \frac{i}{\pi ka}\int_0^1 e^{-2ika\sqrt{1-u^2}} du \; . \tag{19}$$

Letting u = sin ϕ, the last integral in Equ. (19) becomes

$$\int_0^1 e^{-2ika\sqrt{1-u^2}}\,du = \int_0^{\pi/2} e^{-2ika\cos\phi}\cos\phi\,d\phi$$

$$= 1 - \frac{\pi}{2} S_1(2ka) - \frac{i\pi}{2} J_1(2ka)\,. \tag{20}$$

This last integration was done with the help of one integration by parts, and reference (c). S_1 (2ka) denotes the Struve function of order one and argument 2ka. Therefore, we have

$$Q_{00} = \frac{1}{2} + \frac{i}{2ka} S_1(2ka) - \frac{1}{2ka} J_1(2ka) \tag{21}$$

so that finally we get

$$S_{00} = \frac{1}{2} - \frac{J_1(2ka)}{2ka} \tag{22}$$

and

$$T_{00} = \frac{S_1(2ka)}{2ka}\,. \tag{23}$$

If α_0 = 1, and all other α_n = 0, then we have simply the piston with uniform surface velocity. Referring to Equ. (12),

$$R/\rho c A = 2\,S_{00} = 1 - \frac{J_1(2ka)}{ka} \tag{24}$$

and

$$X/\rho c A = 2\,T_{00} = \frac{S_1(2ka)}{ka}\,. \tag{25}$$

These are the familiar equations for the radiation resistance and reactance of a circular piston.

The same methods that were used to evaluate Q_{00} will be used in obtaining Q_{20}.

$$Q_{20} = \int_0^1 Z_2\, x\,dx\,. \tag{26}$$

Referring to reference (a) for Z_2, and again setting x = r/a,

$$Z_2 = x^2 - 1 + \left(1 - \frac{2}{(ka)^2}\right) Z_0$$

$$+ \frac{2i}{\pi ka}\int_0^{\pi} \sqrt{1 - x^2\sin^2\alpha}\; e^{-ika\left(x\cos\alpha + \sqrt{1-x^2\sin^2\alpha}\right)}\,d\alpha\,. \tag{27}$$

Then we have

$$Q_{20} = \int_0^1 (x^2-1)\, x\, dx + \int_0^1 \left(1 - \frac{2}{(ka)^2}\right) Z_0\, x\, dx$$

$$+ \frac{2i}{\pi ka} \int_0^1 \int_0^{\pi} \sqrt{1-x^2\sin^2\alpha}\; e^{-ika\left(x\cos\alpha + \sqrt{1-x^2\sin^2\alpha}\right)}\, x\, d\alpha\, dx. \tag{28}$$

Simplifying and again changing from the x-α plane to the u-v plane,

$$Q_{20} = \frac{1}{4} - \frac{1}{2} + \left(1 - \frac{2}{(ka)^2}\right) Q_{00}$$

$$+ \frac{2i}{\pi ka} \int_0^1 \int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} e^{-ika\left(v + \sqrt{1-u^2}\right)} \sqrt{1-u^2}\, dv\, du \tag{29}$$

so that

$$Q_{20} = -\frac{1}{4} + \left(1 - \frac{2}{(ka)^2}\right) Q_{00}$$

$$+ \frac{2i}{\pi ka} \int_0^1 \sqrt{1-u^2}\; e^{-ika\sqrt{1-u^2}} \left[\int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} e^{-ikav}\, dv \right] du. \tag{30}$$

Carrying out the inner integration, we see that

$$Q_{20} = -\frac{1}{4} + \left(1 - \frac{2}{(ka)^2}\right) Q_{00}$$

$$+ \frac{2i}{\pi ka} \int_0^1 \sqrt{1-u^2}\; e^{-ika\sqrt{1-u^2}} \left[\frac{e^{-ika\sqrt{1-u^2}} - e^{ika\sqrt{1-u^2}}}{-ika} \right] du \tag{31}$$

and

$$Q_{20} = -\frac{1}{4} + \left(1 - \frac{2}{(ka)^2}\right) Q_{00}$$

$$- \frac{2}{\pi (ka)^2} \int_0^1 \sqrt{1-u^2} \left(e^{-2ika\sqrt{1-u^2}} - 1 \right) du. \tag{32}$$

It can be shown that

$$\int_0^1 \sqrt{1-u^2}\, du = \frac{\pi}{4}. \tag{33}$$

By again letting u = s in ϕ, expressing $\cos^2\phi$ as $1-\sin^2\phi$, and using reference (c), we obtain

$$\int_0^1 \sqrt{1-u^2}\, e^{-2ika\sqrt{1-u^2}}\, du = \int_0^{\pi/2} e^{-2ika\cos\phi} \cos^2\phi\, d\phi$$

$$= \frac{\pi}{2}\left[J_0(2ka) - \frac{J_1(2ka)}{2ka} - i\left(S_0(2ka) - \frac{S_1(2ka)}{2ka} \right)\right]. \quad (34)$$

Finally,

$$S_{20} = \frac{1}{4} - \frac{1}{2(ka)^2} - \frac{J_0(2ka)}{(ka)^2} - \frac{((ka)^2-3)\, J_1(2ka)}{2(ka)^3} \quad (35)$$

and

$$T_{20} = \frac{S_0(2ka)}{(ka)^2} + \frac{((ka)^2-3)\, S_1(2ka)}{2(ka)^3}. \quad (36)$$

Considerable arithmetical care must be taken in using the formulae in Table I. The numerical computation of the $S_{k\ell}$ and $T_{k\ell}$ often involves the subtraction of large comparable numbers, which leads to a reduction in significant figures in the result. For this reason, a very accurate Table of Bessel and Struve functions is required, with perhaps seven (7) figures, such as is found in reference (d).

Unfortunately, this work has not led to a formula not involving integrals for the pressure distribution on a circular piston or on a flexural disk. However, it has led to such a formula for the integral of this pressure over the disk.

David T. Porter

DAVID T. PORTER
Mathematician

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(d) G. N. Watson, A Treatise on the Theory of Bessel Functions, 2nd Edition, Macmillan Co., New York, 1948.

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APPENDIX

THE RECIPROCITY OF $Q_{k\ell}$ AND $Q_{\ell k}$

We wish to show that $Q_{k\ell} \equiv Q_{\ell k}$. To do this, we first write $Q_{k\ell}$ as

$$Q_{k\ell} = \int_0^1 Z_k \, x^{\ell+1} \, dx = \frac{1}{a^2} \int_0^a Z_k \, (r/a)^\ell \, r \, dr . \tag{A1}$$

Integrating with respect to an angle α around a circle,

$$Q_{k\ell} = \frac{1}{2\pi a^2} \int_0^{2\pi} \int_0^a Z_k \, (r/a)^\ell \, r \, dr \tag{A2}$$

This double integral can be considered as the integral of $Z_k(r/a)^\ell$ over the circular area S_ℓ . Therefore,

$$Q_{k\ell} = \frac{1}{2\pi a^2} \int_{S_\ell} Z_k (r/a)^\ell \, d S_\ell \tag{A3}$$

and so, we wish to show that

$$\int_{S_\ell} Z_k (r/a)^\ell \, dS_\ell = \int_{S_k} Z_\ell (r/a)^k \, dS_k . \tag{A4}$$

Consider now the mutual radiation impedance between two superimposed circular pistons, k and ℓ , which have normal surface velocities

$$V_{nk} = U_k \, (r/a)^k \, e^{i\omega t} \tag{A5}$$

and

$$V_{n\ell} = U_\ell \, (r/a)^\ell \, e^{i\omega t} . \tag{A6}$$

The mutual radiation impedance between the two pistons, referred to their edge velocities, U_k and U_ℓ , will be, following reference (e), Equ. (8),

$$Z_{k\ell} = \frac{1}{V_k V_\ell^\star} \int_{S_\ell} P_k(r_\ell) \; V_{n\ell}^\star(r_\ell) \, d S_\ell . \tag{A7}$$

Because the reciprocity of $Z_{k\ell}$ and $Z_{\ell k}$ applies here,

$$Z_{k\ell} \equiv Z_{\ell k} \tag{A8}$$

and

$$\frac{1}{V_k V_\ell^\star} \int_{S_\ell} P_k(r_\ell) \, V_{n\ell}^\star(r_\ell) \, dS_\ell = \frac{1}{V_\ell V_k^\star} \int_{S_k} P_\ell(r_k) \, V_{nk}^\star(r_k) \, dS_k . \tag{A9}$$

Here

$$v_\ell = U_\ell e^{i\omega t} , \quad v_k = U_k e^{i\omega t} \tag{A10}$$

and

$$P_\ell = \rho c U_\ell e^{i\omega t} Z_\ell , \quad P_k = \rho c U_k e^{i\omega t} Z_k \tag{A11}$$

Therefore, we have

$$\begin{aligned} &\frac{1}{U_k U_\ell} \int_{S_\ell} \rho c U_k Z_k U_\ell (r/a)^\ell dS_\ell \\ &\quad = \frac{1}{U_\ell U_k} \int_{S_k} \rho c U_\ell Z_\ell U_k (r/a)^k dS_k . \end{aligned} \tag{A12}$$

And so,

$$\int_{S_\ell} Z_k (r/a)^\ell dS_\ell = \int_{S_k} Z_\ell (r/a)^k dS_k \tag{A13}$$

which was needed to be shown to prove $Q_{k\ell} \equiv Q_{\ell k}$.

TABLE I

$$\beta = ka \qquad J_0 = J_0(2ka). \quad J_1 = J_1(2ka).$$

$$S_{00} = \frac{1}{2} - \frac{J_1}{2\beta}$$

$$S_{20} = \frac{1}{4} - \frac{1}{2\beta^2} - \frac{J_1}{2\beta^3}(\beta^2 - 3) - \frac{J_0}{\beta^2}$$

$$S_{22} = \frac{1}{6} - \frac{1}{2\beta^2} - \frac{J_1}{2\beta^5}(\beta^4 - 10\beta^2 + 10) - \frac{J_0}{\beta^4}(2\beta^2 - 5)$$

$$S_{40} = \frac{1}{6} - \frac{1}{\beta^2} + \frac{6}{\beta^4} - \frac{J_1}{2\beta^5}(\beta^4 - 14\beta^2 + 40)$$

$$- \frac{J_0}{\beta^4}(2\beta^2 - 14)$$

$$S_{42} = \frac{1}{8} - \frac{5}{6\beta^2} + \frac{3}{\beta^4} - \frac{J_1}{2\beta^7}(\beta^6 - 25\beta^4 + 140\beta^2 - 140)$$

$$- \frac{J_0}{\beta^6}(3\beta^4 - 32\beta^2 + 70)$$

$$S_{44} = \frac{1}{10} - \frac{1}{\beta^2} + \frac{4}{\beta^4} - \frac{J_1}{2\beta^9}(\beta^8 - 44\beta^6 + 504\beta^4 - 2016\beta^2 + 2016)$$

$$- \frac{J_0}{\beta^8}(4\beta^6 - 80\beta^4 + 504\beta^2 - 1008)$$

TABLE II

$\beta = ka$. $S_0 = S_0(2ka)$ $S_1 = S_1(2ka)$.

$$T_{00} = \frac{S_1}{2\beta}$$

$$T_{20} = \frac{S_1}{2\beta^3}(\beta^2 - 3) + \frac{S_0}{\beta^2}$$

$$T_{22} = \frac{S_1}{2\beta^5}(\beta^4 - 10\beta^2 + 10) + \frac{S_0}{\beta^4}(2\beta^2 - 5) + \frac{20}{3\pi\beta^3}$$

$$T_{40} = \frac{S_1}{2\beta^5}(\beta^4 - 14\beta^2 + 40) + \frac{S_0}{\beta^4}(2\beta^2 - 14) + \frac{8}{3\pi\beta^3}$$

$$T_{42} = \frac{S_1}{2\beta^7}(\beta^6 - 25\beta^4 + 140\beta^2 - 140) + \frac{S_0}{\beta^6}(3\beta^4 - 32\beta^2 + 70)$$

$$+ \frac{16}{\pi\beta^3} - \frac{280}{3\pi\beta^5}$$

$$T_{44} = \frac{S_1}{2\beta^9}(\beta^8 - 44\beta^6 + 504\beta^4 - 2016\beta^2 + 2016)$$

$$+ \frac{S_0}{\beta^8}(4\beta^6 - 80\beta^4 + 504\beta^2 - 1008)$$

$$+ \frac{1}{5\pi\beta^7}(160\beta^4 - 2016\beta^2 + 6720)$$